Bayesian nonparametric comorbidity analysis of psychiatric disorders

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Outline

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Introduction

- Individuals with multiple coexisting diseases.
- 80% of US Medicare spending devoted to patients with 4+ chronic conditions.
- Impact of comorbidity: mortality, quality of life, quality of health care, . . .
- Psychiatry: etiological and treatment implications.

Introduction

Goal

Find out the latent relationship among psychiatric disorders.

Database

NESARC database (*National Epidemiologic Survey on Alcohol and Related Conditions*):

- Samples the U.S. population.
- Around 3K questions and 43K subjects.
- Mainly yes-or-no questions, and some multiple-choice and questions with ordinal answers.

Our approach

Latent feature modeling and Indian buffet process (IBP).

Introduction

Bayesian Nonparametrics

- Unbounded number of degrees of freedom in a model.
- E.g., clustering with unknown number of clusters.
- The posterior distribution chooses the complexity of the model to fit the data.

Introduction

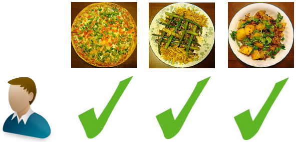
Bayesian Nonparametrics

- Unbounded number of degrees of freedom in a model.
- E.g., clustering with unknown number of clusters.
- The posterior distribution chooses the complexity of the model to fit the data.
- Applications (I've worked on / Would like to work on):
  - Psychiatry.
  - Power disaggregation.
  - Recommendation systems.
  - Multiuser MIMO channel estimation.
  - Channel coding.
  - Sports.
  - NYC marathon modeling.
  - Higgs boson challenge?

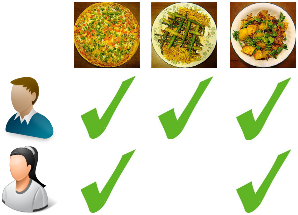
Outline

Indian Buffet Process

Indian Buffet Process



Indian Buffet Process



Indian Buffet Process

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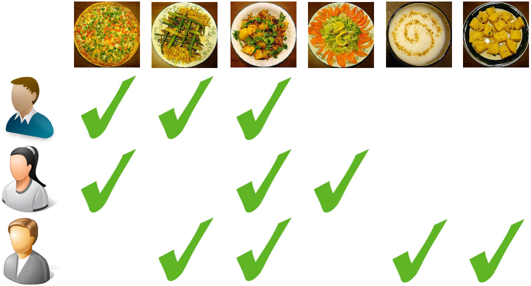

Indian Buffet Process

Indian Buffet Process

⋯

1	1	1	0	0	0
1	0	1	1	0	0
0	1	1	0	1	1

⋮

Indian Buffet Process

1	1	0	1	0	1
1	0	1	0	0	1
0	0	1	0	1	1

. . .

⋮

Indian Buffet Process

- Prior distribution over binary matrices.
- Number of columns (features) $K \to \infty$.
- Matrix $\mathbf{Z}_{N \times K} \sim \mathrm{IBP}(\alpha)$.
- Finite N implies finite number of non-zero columns K_+.

Outline

Observation Model

- Each subject characterized by a binary vector of latent features.

	$\mathbf{z}_n$	x_{nd}
	$[1 \quad 0 \quad 1 \quad 1 \quad \cdots]$	-1.05
	$[0 \quad 0 \quad 0 \quad 1 \quad \cdots]$	0.15
	$[1 \quad 0 \quad 0 \quad 1 \quad \cdots]$	1.35

Observation Model

- Each subject characterized by a binary vector of latent features.
- Under Gaussian observations, features are weighted and added.

	$\mathbf{z}_n$		$\mathbf{b}^d$		x_{nd}
	$[1 \quad 0 \quad 1 \quad 1 \quad \cdots]$				-1.05
	$[0 \quad 0 \quad 0 \quad 1 \quad \cdots]$	$\times$	$\begin{bmatrix} 1.2 \\ 3.2 \\ -2.4 \\ 0.15 \\ \vdots \end{bmatrix}$	$=$	0.15
	$[1 \quad 0 \quad 0 \quad 1 \quad \cdots]$				1.35

Observation Model

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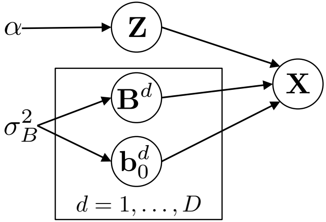
$$
\begin{array}{ccccc}
\mathbf{z}_n & & \mathbf{b}^d & & x_{nd} \\
[1 \quad 0 \quad 1 \quad 1 \quad \cdots] & & \left[\begin{array}{c} 1.2 \\ 3.2 \\ -2.4 \\ 0.15 \\ \vdots \end{array}\right] & & -1.05 \\
[0 \quad 0 \quad 0 \quad 1 \quad \cdots] & \times & & = & 0.15 \\
[1 \quad 0 \quad 0 \quad 1 \quad \cdots] & & & & 1.35
\end{array}
$$

$$\mathbf{ZB} + \text{noise} = \mathbf{X}$$

Observation Model

- Categorical observations: $x_{nd} \in \{1, \ldots, R\}$ ('Yes', 'No', 'Unkown', 'Blank', . . .).
- How to link latent features $\mathbf{z}_{n\cdot}$ and observations?



Observation Model

- Multiple-logistic (softmax) function:

$$p(x_{nd} = \text{'yes'} | \mathbf{z}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) \propto \exp(\quad \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot\text{yes}}^d),$$
$$p(x_{nd} = \text{'no'} | \mathbf{z}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) \propto \exp(\quad \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot\text{no}}^d),$$
$$\cdots$$

$$p(x_{nd} = r | \mathbf{z}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) = \frac{\exp(\quad \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot r}^d)}{\sum_{r'=1}^{R} \exp(\quad \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot r'}^d)}, \quad r = 1, \ldots, R.$$

- The weighting factors are placed Gaussian priors.

Observation Model

- Multiple-logistic (softmax) function:

$$p(x_{nd} = \text{'yes'} | \mathbf{z}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) \propto \exp\left(b_{0\text{yes}}^d + \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot\text{yes}}^d\right),$$
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$$\cdots$$

$$p(x_{nd} = r | \mathbf{z}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) = \frac{\exp\left(b_{0r}^d + \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot r}^d\right)}{\sum_{r'=1}^{R} \exp\left(b_{0r'}^d + \mathbf{z}_{n\cdot}\mathbf{b}_{\cdot r'}^d\right)}, \quad r = 1, \ldots, R.$$

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Inference

- The model is not conditionally conjugate.
- Posterior on the weighting factors:

$$\overbrace{p(\mathbf{B}^d, \mathbf{b}_0^d | \mathbf{X}, \mathbf{Z})}^{\text{Non Gauss}} = \frac{\overbrace{p(\mathbf{x}_{\cdot d} | \mathbf{B}^d, \mathbf{b}_0^d, \mathbf{Z})}^{\text{Non Gauss}} \overbrace{p(\mathbf{B}^d) p(\mathbf{b}_0^d)}^{\text{Gauss}}}{p(\mathbf{x}_{\cdot d} | \mathbf{Z})}.$$

- The integral in the denominator is intractable:

$$p(\mathbf{x}_{\cdot d} | \mathbf{Z}) = \int p(\mathbf{x}_{\cdot d} | \mathbf{B}^d, \mathbf{b}_0^d, \mathbf{Z}) p(\mathbf{B}^d) p(\mathbf{b}_0^d) d\mathbf{B}^d d\mathbf{b}_0^d.$$

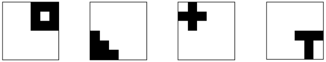
Gibbs sampling

Gibbs sampling

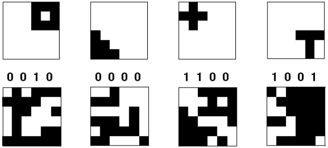
- Iteratively sample each element of the IBP matrix.
- Integrate out all the weighting factors $\Rightarrow$ Intractable.
- Gaussian approximation of the posterior:
  1. Laplace approximation.
  2. Expectation propagation.
     - Multinomial probit likelihood instead of softmax.

Outline

Toy Example



Toy Example



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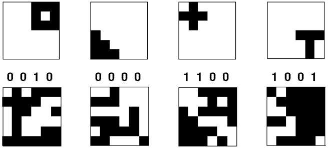
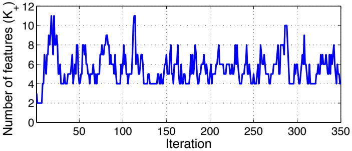
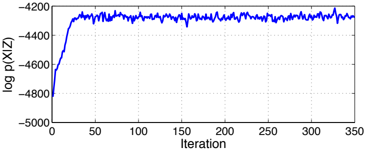
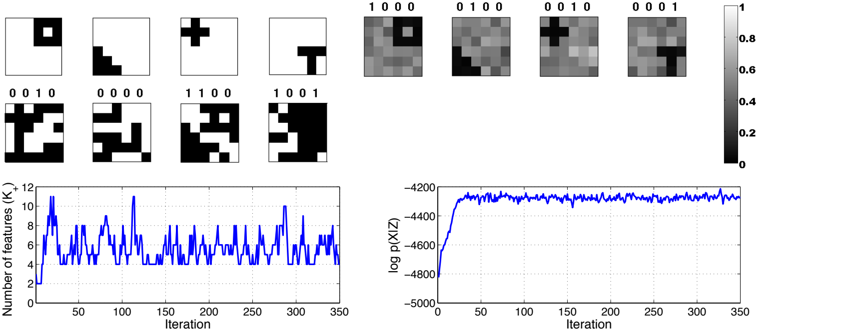

0 0 1 0
0 0 0 0
1 1 0 0
1 0 0 1

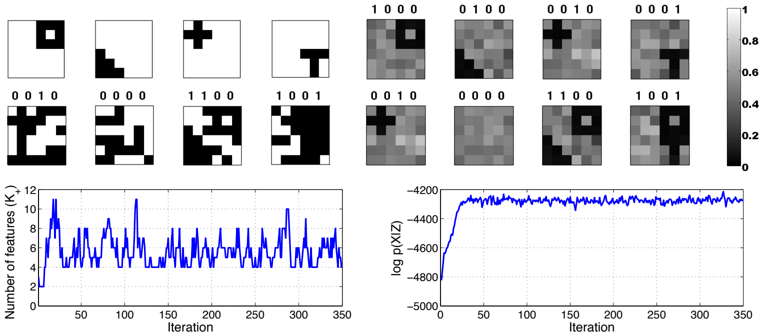

Number of features (K+)
Iteration
0
2
4
6
8
10
12
50
100
150
200
250
300
350


log p(X|Z)
Iteration
−4200
−4400
−4600
−4800
−5000
0
50
100
150
200
250
300
350

Toy Example


0 0 1 0
0 0 0 0
1 1 0 0
1 0 0 1
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
1
0.8
0.6
0.4
0.2
0
Number of features (K_+)
Iteration
log p(X|Z)
Iteration

Toy Example

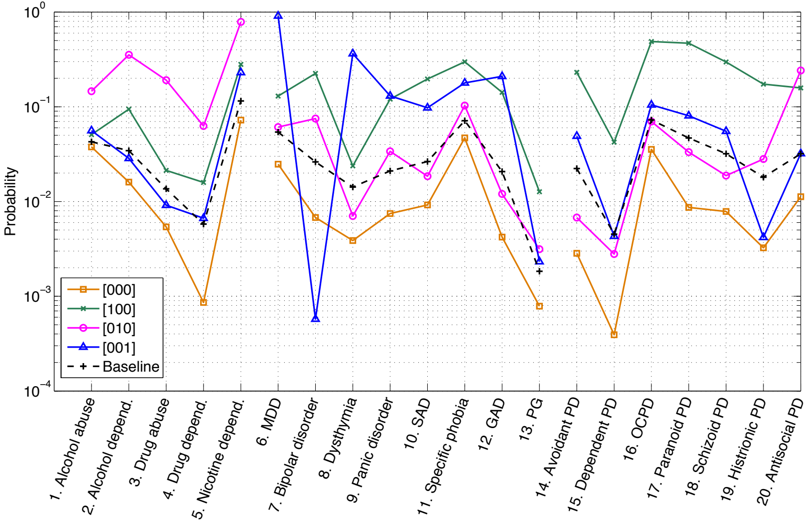

1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
0 0 1 0
0 0 0 0
1 1 0 0
1 0 0 1
0 0 1 0
0 0 0 0
1 1 0 0
1 0 0 1
1
0.8
0.6
0.4
0.2
0
Number of features (K_+)
Iteration
log p(X|Z)
Iteration

Experiments on NESARC

Real data

- Inputs: Diagnoses of 20 common psychiatric disorders.
- Previous studies: Factor analysis.
  - Specify the number of factors.
  - Assume Gaussian observations.
  - 3 latent factors seem enough.

Experiments on NESARC



Experiments on NESARC

Extension of the model

- Individual-specific severity terms:

$$p(x_{nd} = r | \mathbf{w}_{n\cdot}, \mathbf{b}_0^d, \mathbf{B}^d) \propto \exp\left(b_{0r}^d + \mathbf{w}_{n\cdot}\mathbf{b}_{\cdot r}^d\right), \quad r = 1, \ldots, R.$$

- Instead of on/off features, each term in $[0, 1]$.

Experiments on NESARC

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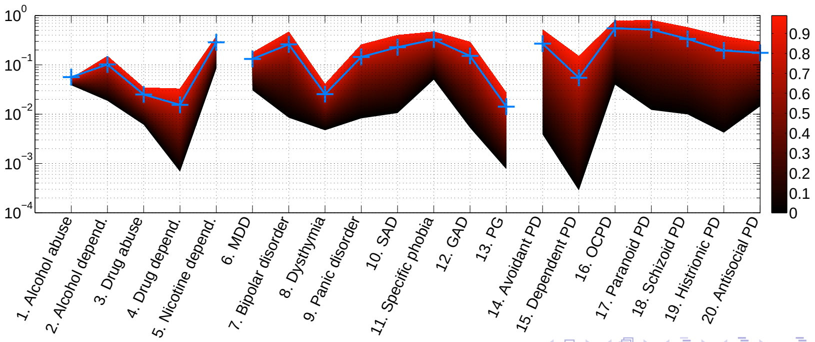

Outline

Conclusions

Summary

- Likelihood model for the IBP with categorical observations.

Inference Algorithm	Likelihood
Gibbs sampling + Laplace approximation	Softmax
Gibbs sampling + Nested EP	Multinomial probit
Variational inference	Softmax

- Results are in agreement with previous work and also provide new information.

Outline

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