

BAYESIAN NONPARAMETRICS FOR TIME SERIES MODELING

DOCTORAL THESIS

Francisco Jesús Rodríguez Ruiz



June 30th, 2015

OUTLINE

① INTRODUCTION

② BAYESIAN NONPARAMETRICS

③ CONTRIBUTIONS

- Infinite Factorial Unbounded-State HMM
- Infinite Factorial Finite State Machine

④ CONCLUSIONS

OUTLINE

① INTRODUCTION

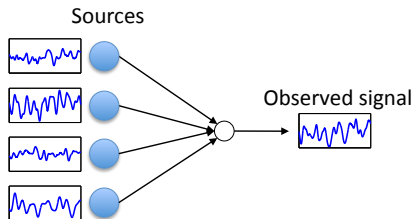
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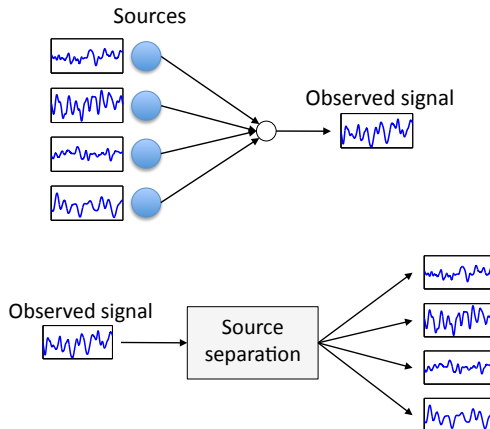
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④ CONCLUSIONS

MOTIVATION



MOTIVATION



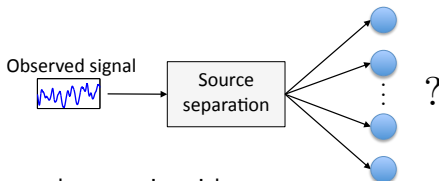
MOTIVATION

Applications:

- Power disaggregation.
- Multiuser communication systems.
- Speech separation.
- Multi-target tracking.
- Electroencephalography (EEG).
- ...

MOTIVATION

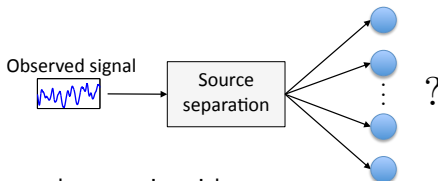
How many hidden sources?



- Classical approaches require either
 - known number of sources.
 - upper bound.
 - model selection.

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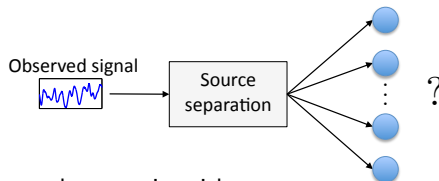
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- Bayesian nonparametrics can
 - infer the number of latent sources from the data.
 - avoid model selection.

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OUR APPROACH

Bayesian nonparametric modeling of
discrete-time series for source separation problems

WHY BNP?



WHY BNP?



- Pick a large enough #sources.

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- Model selection (AIC, BIC).

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- Bayesian model selection.

WHY BNP?



- Pick a large enough #sources.
- Model selection (AIC, BIC).
- Bayesian model selection.
- BNP:
 - Model complexity grows with data size.
 - Unbounded #sources.

STATE OF THE ART

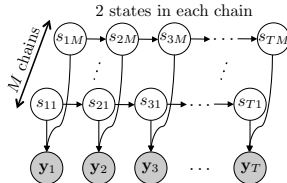
- Many BNP models for discrete-time series.
 - e.g., infinite HMM.
- Not many BNP models for source separation.

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- Many BNP models for discrete-time series.
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- Not many BNP models for source separation.
 - Infinite ICA.

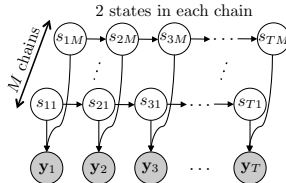
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 - Infinite Factorial HMM (IFHMM).



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- ICA-IFHMM.

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Lack of BNP models for source separation:

- Infinite factorial HMM with **non-binary** hidden states.
 - e.g., power disaggregation.

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- Model that accounts for **multipath propagation**.
 - e.g., multiuser communications systems.

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Lack of BNP models for source separation:

- Infinite factorial HMM with **non-binary** hidden states.
 - e.g., power disaggregation.
- Model that accounts for **multipath propagation**.
 - e.g., multiuser communications systems.
- Model with continuous-valued states that captures **temporal dependencies**.
 - e.g., speech separation.

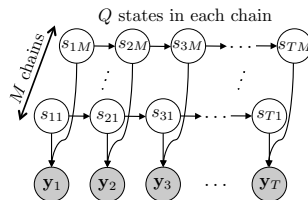
CONTRIBUTIONS

INFINITE FACTORIAL UNBOUNDED-STATE HMM

- ① Non-binary IFHMM.
 - Can infer the number of HMMs in a factorial model.
- ② IFUHMM.
 - Can additionally infer the cardinality of the state space.

Applications:

- Power disaggregation.
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INFINITE FACTORIAL FINITE STATE MACHINE

- Can infer the number of FSMs in a factorial model.
- Naturally account for multipath, echo, ...

Applications:

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BAYESIAN NONPARAMETRICS

- Bayesian framework for **model selection**.
- Prior over **infinite-dimensional** parameter space.
- Only a **finite subset** of the parameters is used for any finite dataset.
- The model complexity is allowed to grow with data size.
- Rely on **stochastic processes**:
 - Gaussian process.
 - Dirichlet process.
 - Beta process.
 - ...

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INDIAN BUFFET PROCESS

- Prior over binary matrices with infinite columns.
- Rows \equiv Data points. Columns \equiv Features.
- $\mathbf{S} \sim \text{IBP}(\alpha)$.
- α : Concentration parameter.
- Each element $s_{tm} \in \{0, 1\}$ indicates whether the m -th feature contributes to the t -th data point.
- Only a finite number of columns M_+ active for any finite number of rows.

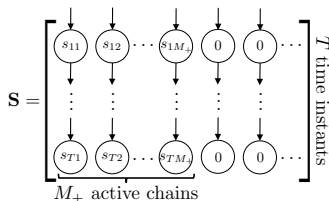
$$\mathbf{S} = \underbrace{\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{bmatrix}}_{\substack{M_+ \text{ non-zero columns} \\ M \text{ columns (features)}}} \begin{matrix} T \\ \text{observations} \end{matrix}$$

MARKOV INDIAN BUFFET PROCESS

- Prior over binary matrices with infinite columns.
- Each column follows a Markov process.
- For any T , only M_+ chains become active.
- The probability $p(\mathbf{S})$ vanishes, but $p([\mathbf{S}]) > 0$.
 - $[\mathbf{S}]$: set of matrices equivalent to \mathbf{S} .
- Useful to build a (binary) infinite factorial HMM.

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{bmatrix} \begin{matrix} T \text{ time instants} \\ \\ \\ \end{matrix}$$

$\underbrace{\hspace{10em}}_{M_+ \text{ non-zero columns}}$
 $\underbrace{\hspace{10em}}_{M \text{ columns (chains)}}$



MARKOV INDIAN BUFFET PROCESS

$$\mathbf{S} \sim \text{MIBP}(\alpha, \beta_0, \beta_1)$$

- Can be obtained by defining the transition probabilities

$$\mathbf{A}^m = \begin{bmatrix} a^m & 1 - a^m \\ b^m & 1 - b^m \end{bmatrix} \quad \begin{aligned} a^m &= p(s_{tm} = 0 | s_{(t-1)m} = 0) \\ b^m &= p(s_{tm} = 0 | s_{(t-1)m} = 1) \end{aligned}$$

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- ... with priors

$$a^m \sim \text{Beta}\left(1, \frac{\alpha}{M}\right) \quad b^m \sim \text{Beta}(\beta_0, \beta_1)$$

- ... and let $M \rightarrow \infty$

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- ... and let $M \rightarrow \infty$
- After integrating out a^m and b^m :

$$\lim_{M \rightarrow \infty} p([\mathbf{S}]) = \frac{\alpha^{M_+}}{\prod_{h=1}^{2T} M_h!} e^{-\alpha H_T} \prod_{m=1}^{M_+} \frac{(n_{01}^m - 1)!(n_{00}^m)! \Gamma(\beta_0 + \beta_1) \Gamma(\beta_0 + n_{10}^m) \Gamma(\beta_1 + n_{11}^m)}{(n_{00}^m + n_{01})! \Gamma(\beta_0) \Gamma(\beta_1) \Gamma(\beta_0 + \beta_1 + n_{10}^m + n_{11}^m)}$$

- Markov exchangeable in the rows.

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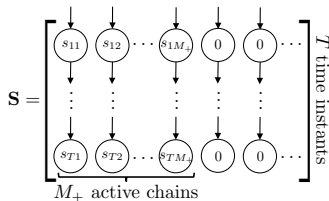
④ CONCLUSIONS

NON-BINARY INFINITE FACTORIAL HMM

- Generalization of the MIBP for non-binary matrices.
- Each state $s_{tm} \in \{0, 1, \dots, Q - 1\}$.
- Inactive state ($s_{tm} = 0$).

$$\mathbf{S} = \left[\begin{array}{ccccccc} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{array} \right] \begin{array}{l} T \text{ time instants} \\ \\ \\ \end{array}$$

$\underbrace{\hspace{10em}}_{M_+ \text{ non-zero columns}}$
 $\underbrace{\hspace{10em}}_{M \text{ columns (chains)}}$



NON-BINARY INFINITE FACTORIAL HMM

- Can be obtained by defining the transition probabilities

$$\mathbf{A}^m = \begin{bmatrix} a_{00}^m & a_{01}^m & \cdots & a_{0(Q-1)}^m \\ a_{10}^m & a_{11}^m & \cdots & a_{1(Q-1)}^m \\ \vdots & \vdots & \ddots & \vdots \\ a_{(Q-1)0}^m & a_{(Q-1)1}^m & \cdots & a_{(Q-1)(Q-1)}^m \end{bmatrix}$$

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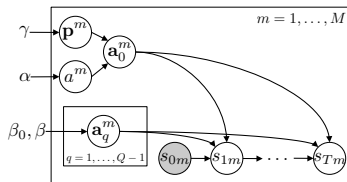
$$\mathbf{A}^m = \begin{bmatrix} a_{00}^m & a_{01}^m & \cdots & a_{0(Q-1)}^m \\ a_{10}^m & a_{11}^m & \cdots & a_{1(Q-1)}^m \\ \vdots & \vdots & \ddots & \vdots \\ a_{(Q-1)0}^m & a_{(Q-1)1}^m & \cdots & a_{(Q-1)(Q-1)}^m \end{bmatrix}$$

- Prior distribution:

$$a^m \sim \text{Beta}\left(1, \frac{\alpha}{M}\right) \quad \mathbf{p}^m \sim \text{Dirichlet}(\gamma)$$

$$\mathbf{a}_0^m = [a^m \quad (1 - a^m)\mathbf{p}^m]$$

$$\mathbf{a}_q^m \sim \text{Dirichlet}(\beta_0, \beta, \dots, \beta), \quad q = 1, \dots, Q-1$$



LIMIT OF $p([\mathbf{S}])$

$$\begin{aligned}
 \lim_{M \rightarrow \infty} p([\mathbf{S}]) &= \frac{(Q-1)!}{(Q-N_Q)!N_f} \frac{\alpha^{M_+}}{Q^{T-1} \prod_{h=1} M_h!} e^{-\alpha H_T} \\
 &\times \prod_{m=1}^{M_+} \left[\frac{\Gamma(n_{00}^m + 1) \Gamma\left(\sum_{i=1}^{Q-1} n_{0i}^m\right)}{\Gamma(n_{0\bullet}^m + 1)} \frac{\Gamma((Q-1)\gamma) \prod_{i=1}^{Q-1} \Gamma(n_{0i}^m + \gamma)}{\Gamma\left(\sum_{i=1}^{Q-1} (n_{0i}^m + \gamma)\right) (\Gamma(\gamma))^{Q-1}} \right. \\
 &\quad \left. \times \prod_{q=1}^{Q-1} \left(\frac{\Gamma(\beta_0 + (Q-1)\beta)}{\Gamma(\beta_0) (\Gamma(\beta))^{Q-1}} \frac{\Gamma(n_{q0}^m + \beta_0) \prod_{i=1}^{Q-1} \Gamma(n_{qi}^m + \beta)}{\Gamma(n_{q\bullet}^m + \beta_0 + (Q-1)\beta)} \right) \right].
 \end{aligned}$$

CULINARY METAPHOR

$Q = 3$ states (1 inactive + 2 active)

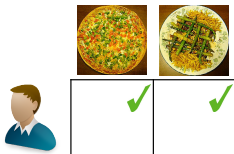
$t = 1$



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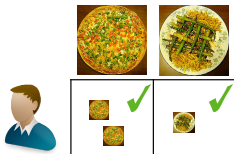
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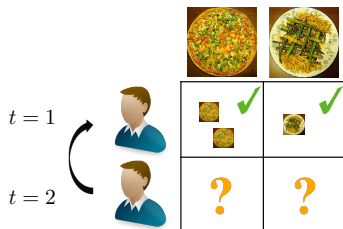
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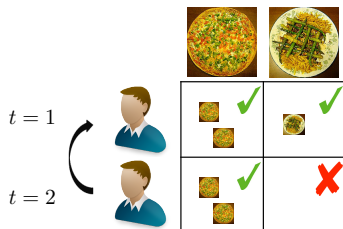
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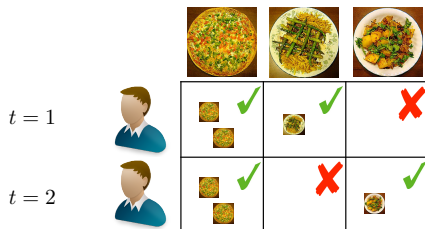
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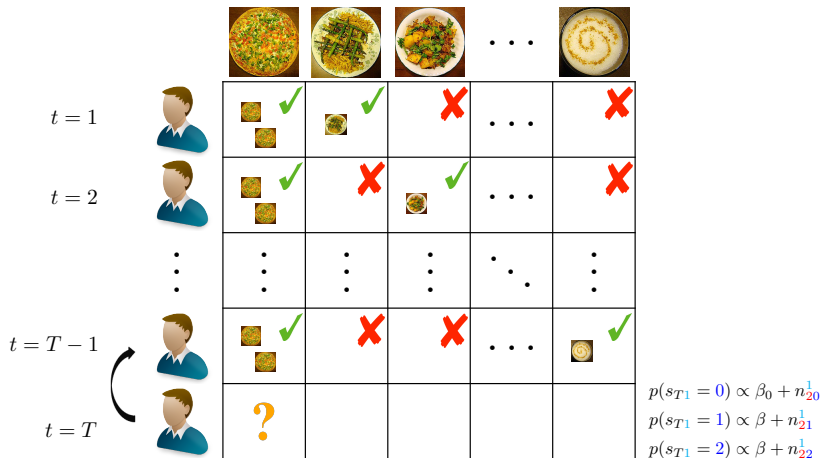
CULINARY METAPHOR

$Q = 3$ states (1 inactive + 2 active)

					...	
$t = 1$		 ✓	 ✓	✗	...	✗
$t = 2$		 ✓	✗	 ✓	...	✗
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$t = T - 1$		 ✓	✗	✗	...	 ✓

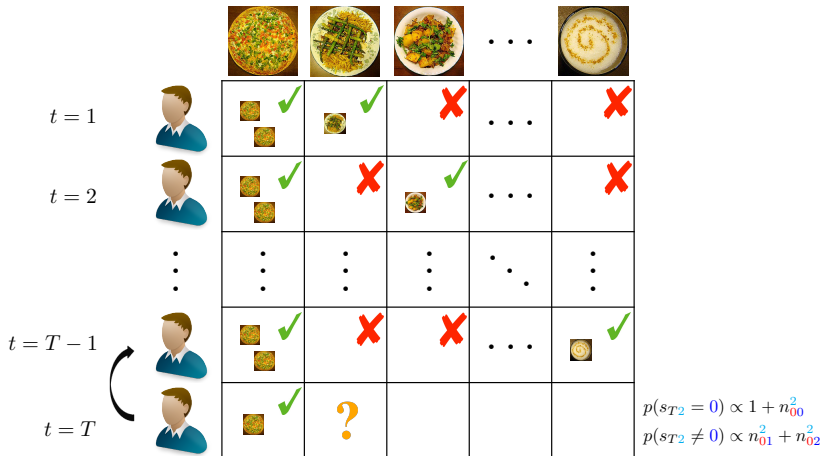
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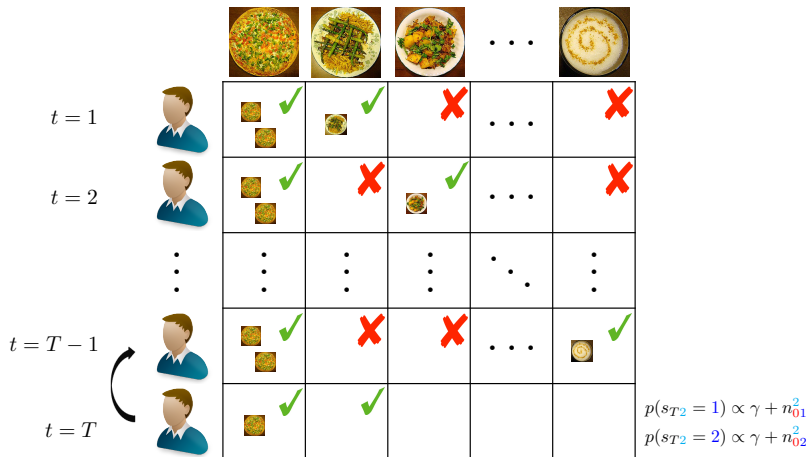
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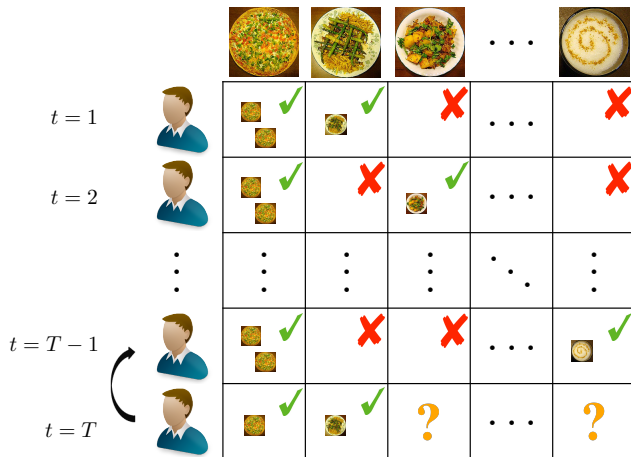
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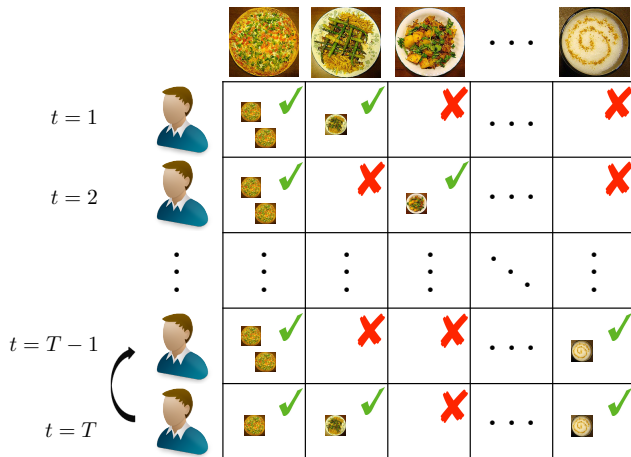
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					...		
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\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots	
$t = T - 1$		 ✓	✗	✗	...	 ✓	✗
$t = T$		 ✓	 ✓	✗	...	 ✓	 ✓










CULINARY METAPHOR

$Q = 3$ states (1 inactive + 2 active)

					...		
$t = 1$		2	1	0	...	0	0
$t = 2$		2	0	1	...	0	0
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
$t = T - 1$		2	0	0	...	1	0
$t = T$		1	1	0	...	1	2










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					...		
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$t = T$		0	1	2	...	1	1

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					...		
$t = 1$		0	1	0	...	2	0
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\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
$t = T - 1$		0	1	0	...	0	2
$t = T$		0	2	1	...	2	2

INFERENCE

FIXED Q

① MCMC:

- Sample from the posterior.
- Blocked sampling approach.
- Slice sampling \rightarrow
Stick-breaking construction.
- FFBS for each Markov chain.

② Variational:

- Approximate the posterior.
- Structured approach.
- Involves a
forward-backward
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INFINITE FACTORIAL UNBOUNDED-STATE HMM

- Prior over the number of states:

$$Q = 2 + Q', \quad Q' \sim \text{Poisson}(\lambda)$$

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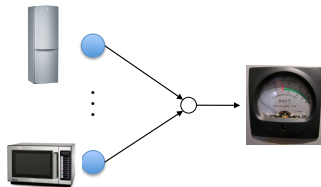
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UNKNOWN Q

- ① MCMC:
 - Based on reversible jump MCMC.
 - Integrate out dimension-changing variables.
 - Updating variables:
 - Q : Split/merge, birth/death.
 - M_+ : Slice sampling.

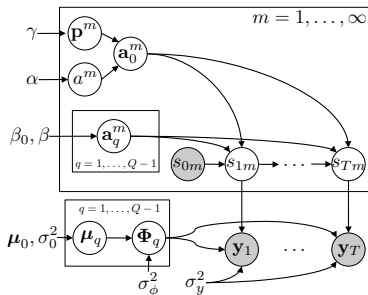
POWER DISAGGREGATION

- Estimate the power consumption of each device.
- Non-invasive measurements.
 - Improve efficiency of consumers.
 - Detect faulty equipment.
- Two datasets.
 - REDD (1 day, 5 houses, 6 devices).
 - AMP (2 days, 1 house, 8 devices).



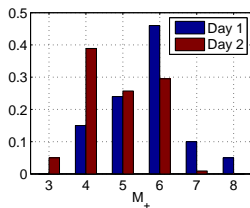
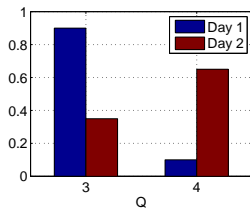
POWER DISAGGREGATION

Gaussian observation model



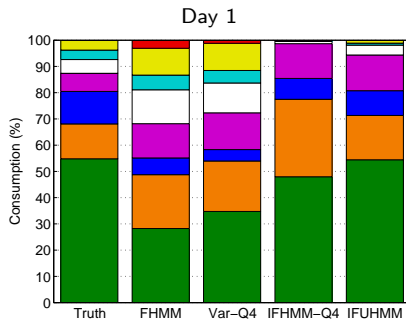
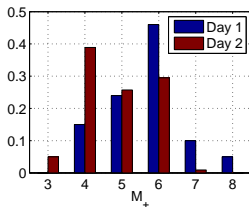
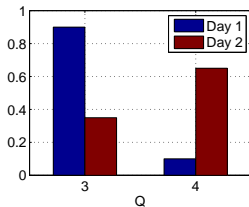
POWER DISAGGREGATION

Results for the AMP database (2 days, 8 devices):



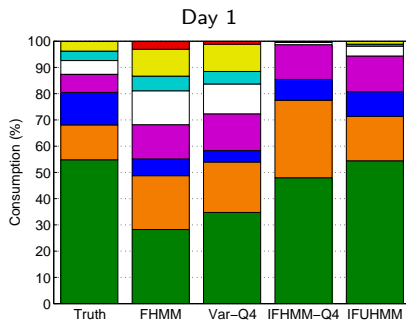
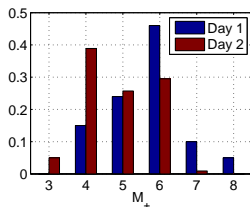
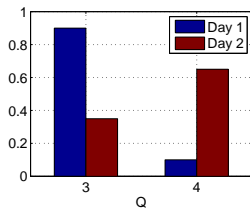
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POWER DISAGGREGATION

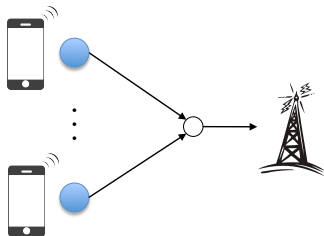
Results for the AMP database (2 days, 8 devices):



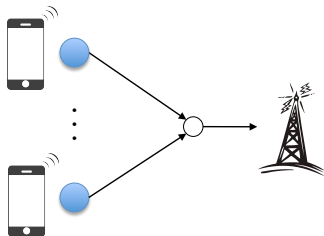
$$\text{accuracy} = 1 - \frac{\sum_{t=1}^T \sum_{m=1}^M |y_t^{(m)} - \hat{y}_t^{(m)}|}{2 \sum_{t=1}^T \sum_{m=1}^M y_t^{(m)}}$$

FHMM	Var-Q4	IFHMM-Q4	IFUHMM
0.36 ± 0.05	0.48 ± 0.06	0.58 ± 0.11	0.69 ± 0.10

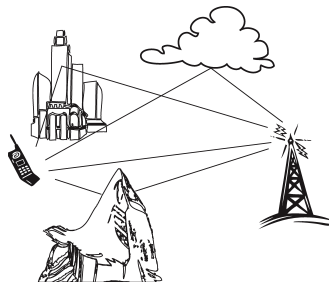
MULTIUSER COMMUNICATION SYSTEM



MULTIUSER COMMUNICATION SYSTEM



Multipath propagation



OUTLINE

① INTRODUCTION

② BAYESIAN NONPARAMETRICS

③ CONTRIBUTIONS

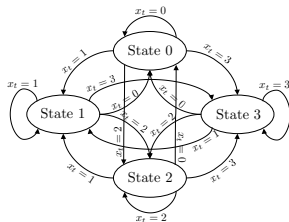
Infinite Factorial Unbounded-State HMM

Infinite Factorial Finite State Machine

④ CONCLUSIONS

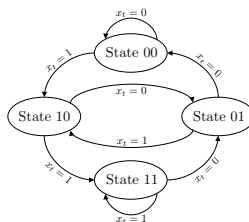
FINITE-MEMORY FINITE STATE MACHINE

Finite-Memory FSM: The state depends on the last L inputs x_t .



HMM with $Q = 4$ states.

Dense transition probability matrix.

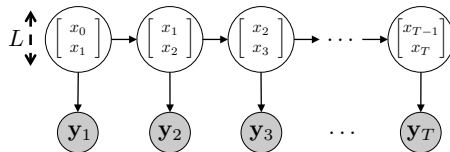


FSM with memory length $L = 2$.

Sparse transition probability matrix.

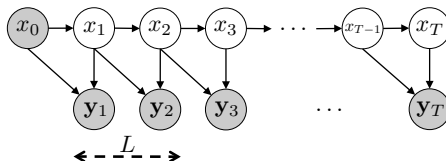
INFINITE FACTORIAL FINITE STATE MACHINE

- HMM representation of an FSM:



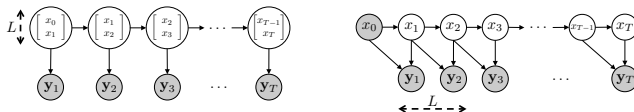
State space cardinality: $|\mathcal{X}|^L$.

- Alternative representation (likelihood accounts for the memory):



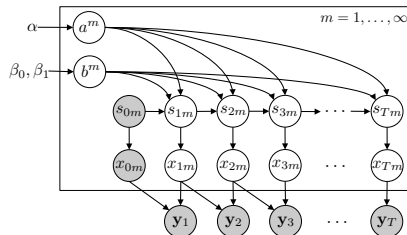
INFINITE FACTORIAL FINITE STATE MACHINE

- The likelihood accounts for the memory.



- Infinite Factorial FSM:

- $M \rightarrow \infty$ parallel FSMs.
- $\mathbf{S} \sim \text{MIBP}(\alpha, \beta_0, \beta_1)$.
- Auxiliary variables s_{tm} indicate activity/inactivity.
- $x_{tm} = 0$ if $s_{tm} = 0$ and $x_{tm} \in \mathcal{A}$ otherwise.



INFERENCE

INFERENCE

MCMC inference algorithm:

- ① Propose new parallel FSMs.
 - Slice sampling.
 - Stick-breaking construction.
- ② Update hidden states x_{tm} , s_{tm} .
 - Particle Gibbs with ancestor sampling.
- ③ Remove inactive FSMs.
- ④ Sample global variables.

INFERENCE

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MCMC inference algorithm:

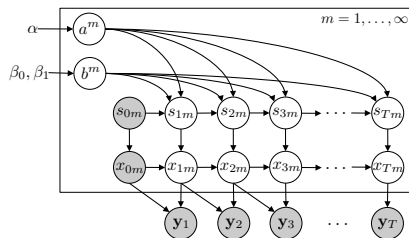
- ① Propose new parallel FSMs.
 - Slice sampling.
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- ② Update hidden states x_{tm} , s_{tm} .
 - Particle Gibbs with ancestor sampling.
- ③ Remove inactive FSMs.
- ④ Sample global variables.

PARTICLE GIBBS WITH ANCESTOR SAMPLING

- Combines MCMC and SMC.
- Better mixing properties than FFBS.
- Outperforms FFBS:
 - Quadratic complexity with memory L .
 - Can handle more general models.

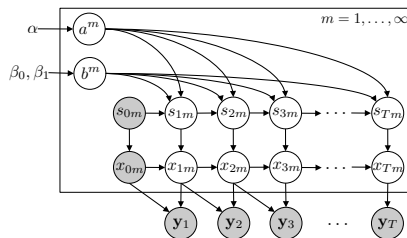
GENERALIZATION OF THE MODEL

- Extensions that we can easily handle:
 - States x_{tm} do not necessarily belong to finite set.
 - The state x_{tm} depends on $x_{(t-1)m}$.



GENERALIZATION OF THE MODEL

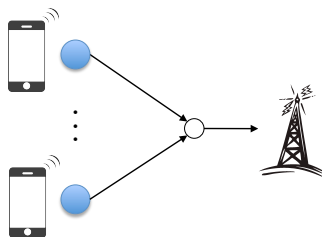
- Extensions that we can easily handle:
 - States x_{tm} do not necessarily belong to finite set.
 - The state x_{tm} depends on $x_{(t-1)m}$.



- Applications:
 - Multi-target tracking.
 - Speech separation.
 - ...

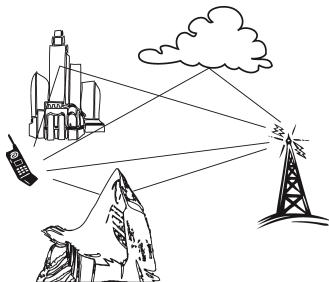
MULTIUSER COMMUNICATION SYSTEM

- Estimate the number of users and the transmitted symbols.
- Machine-to-machine communications:
 - Transmitters switching on and off asynchronously.
 - Short bursts of symbols.
 - Reduce message overhead.
 - 5G systems.



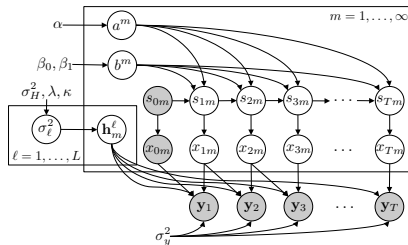
MULTIUSER COMMUNICATION SYSTEM

Multipath propagation



Gaussian observation model

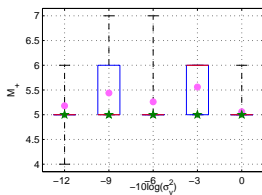
$$\mathbf{y}_t = \sum_{m=1}^{M_+} \sum_{\ell=1}^L \mathbf{h}_m^\ell x_{(t-\ell+1)m} + \mathbf{n}_t$$



MULTIUSER COMMUNICATION SYSTEM

Synthetic experiment with 5 transmitters and 20 receivers.

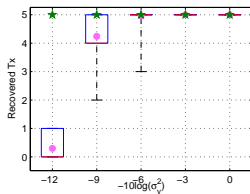
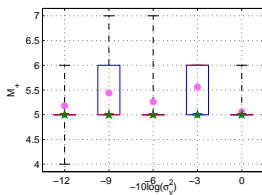
$$L = 1$$



MULTIUSER COMMUNICATION SYSTEM

Synthetic experiment with 5 transmitters and 20 receivers.

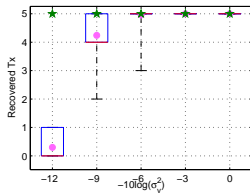
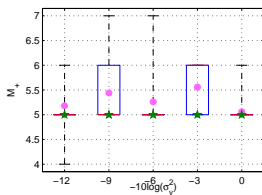
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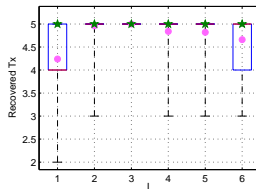
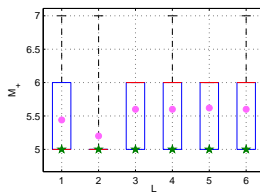
MULTIUSER COMMUNICATION SYSTEM

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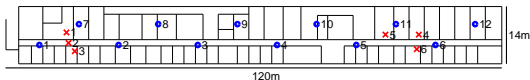
Varying L (-9 dB)



MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

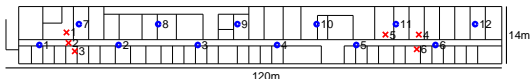
- Ray-tracing software (WISE).
- 6 transmitters, 12 receivers.
- Office at Bell Labs Crawford Hill.



MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

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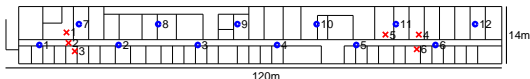
- Recovered transmitters / Inferred M_+ :

Algorithm	L				
	1	2	3	4	5
PGAS	6/6	6/6	6/6	6/6	6/6
FFBS	3/11	3/11	3/8	1/10	—

MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

- Ray-tracing software (WISE).
- 6 transmitters, 12 receivers.
- Office at Bell Labs Crawford Hill.



- Recovered transmitters / Inferred M_+ :

Algorithm	L				
	1	2	3	4	5
PGAS	6/6	6/6	6/6	6/6	6/6
FFBS	3/11	3/11	3/8	1/10	—

- MSE ($\times 10^{-6}$) of the first channel tap ($\ell = 1$):

Algorithm	L				
	1	2	3	4	5
PGAS	2.58	2.51	0.80	0.30	0.16
FFBS	2.79	1.38	5.53	1.90	—

(noise variance is $\sim 10^{-8}$)

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- Infinite Factorial Finite State Machine

④ CONCLUSIONS

CONCLUSIONS

CONTRIBUTIONS

- ① Non-Binary Infinite Factorial HMM.
 - MCMC inference.
 - Variational inference.
- ② Infinite Factorial Unbounded-State HMM.
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 - Particle MCMC inference.

CONCLUSIONS

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- ❶ Non-Binary Infinite Factorial HMM.
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 - MCMC inference.
- ❸ Infinite Factorial Finite State Machine.
 - Particle MCMC inference.

FUTURE WORK

- Doubly nonparametric IFHMM.
- Semi-Markov approaches.
- Inference:
 - Scalability.
 - Mixing of MCMC.
 - Online.
- Other applications.
- Time-varying channels.

Thanks for your attention!



BINARY IFHMM FOR POWER DISAGGREGATION

- REDD dataset (5 houses, 1 day, 6 devices).
- Binary IFHMM ($Q = 2$).
- Histogram of inferred M_+ :

